CoE: Deep Coupled Embedding for Non-Rigid Point Cloud Correspondences

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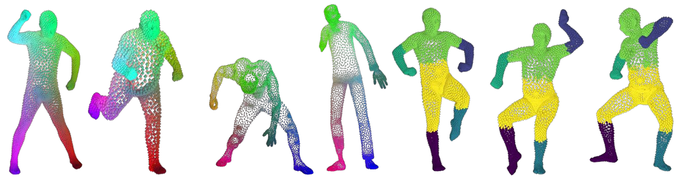
Overview

We propose **CoE**, an unsupervised learning method to learn **geometry-aware** coupled embeddings for non-rigidly shape matching.



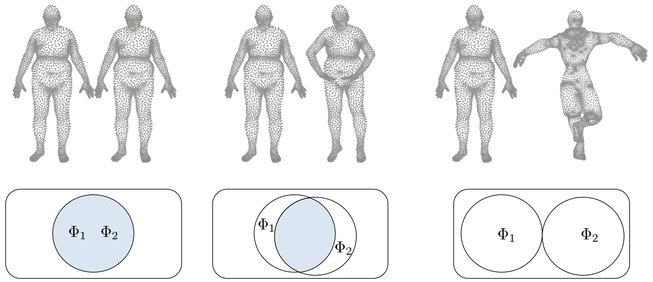
Coupled LBO

Laplace-Beltrami Operator (LBO) Eigenfunction:

- ✓ Spectral Ordering by Eigenvalues
- ✓ Invariance under Isometries
- × Sensitive to non-isometric deformations
- × Unstable under numerical instability

Coupled LBO:

- ✓ Approximation of LBO eigenfunction w/o instability
- ✓ Invariance under non-isometric shape pair
- ✓ Spanning a common spectral space for shapes

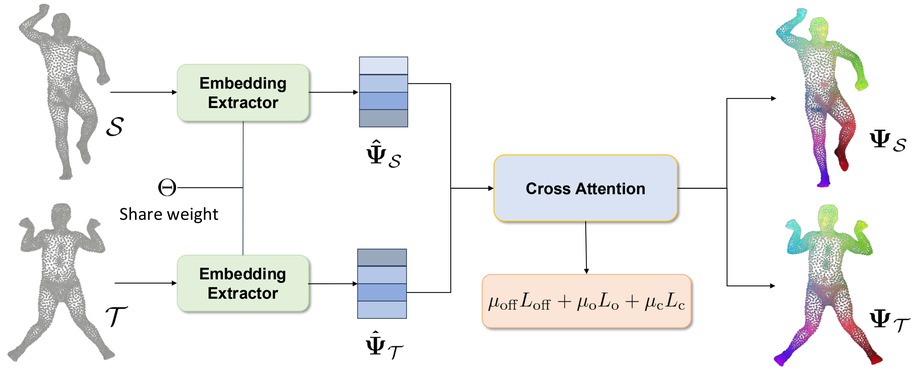


Same Shapes (Near) Isometric Shapes Non-Isometric Shapes

Enable non-isometric shapes to have **the same spectral space properties** as same shapes

Deep Coupled Embedding

- ✓ DiffusionNet[1]-based **embedding extractor** for geometric consistency
- ✓ **Cross-attention module** for embedding communication
- ✓ Unsupervised Loss Inspired by **Classical Geometry Processsing** [2]



$$L_{\text{off}} = \sum_{i\in\{\mathcal{S},\mathcal{T}\}} \left\| \mathbf{\Psi}_i^\top \mathbf{L}_i \mathbf{\Psi}_i - \mathbf{\Lambda}_i \right\|_F$$

$$L_{\text{o}} = \sum_{i\in\{\mathcal{S},\mathcal{T}\}} \left\| \mathbf{\Psi}_i^\top \mathbf{M}_i \mathbf{\Psi}_i - \mathbf{I} \right\|_F$$

$$L_{\text{c}} = \left\| \mathbf{D}_{\mathcal{S}}^T \mathbf{M}_{\mathcal{S}} \mathbf{\Psi}_{\mathcal{S}} - \mathbf{D}_{\mathcal{T}}^T \mathbf{M}_{\mathcal{T}} \mathbf{\Psi}_{\mathcal{T}} \right\|_F$$

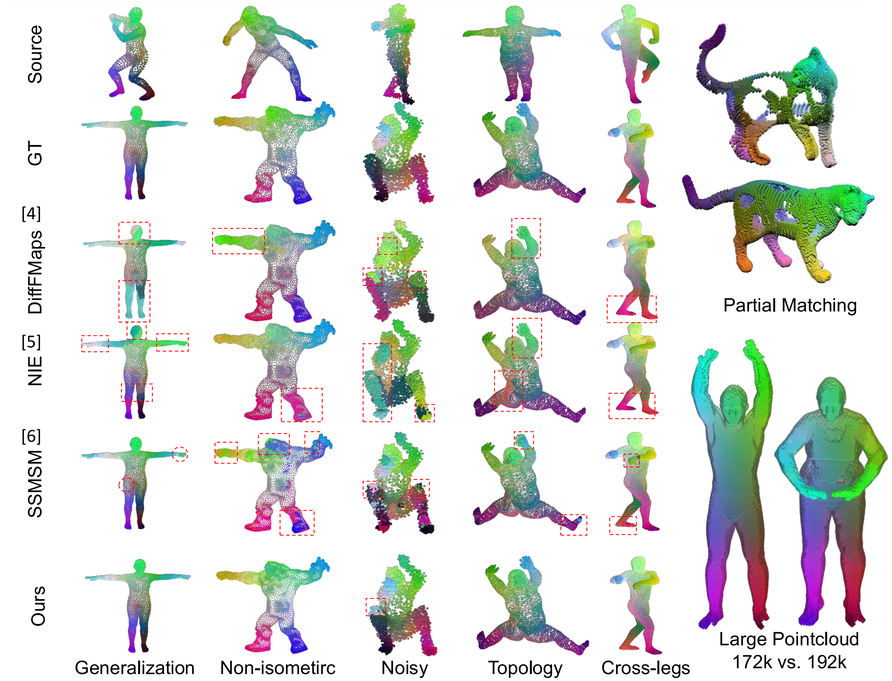
$\mathbf{\Psi}$: Coupled embedding
$\mathbf{L}$: Stiffness matrix
$\mathbf{M}$: Mass matrix
$\mathbf{D}$: Pointwise descriptor
$\mathbf{\Lambda}$: LBO Eigenvalue

Off-diagonal Loss L_{off} : ensure embedding **frequency-aligned**
Orthogonal Loss L_{o} : enforce **basis structure**
Contrastive Loss L_{c} : couple embedding **"speak the same language"**

LBO Eigenfunction [3] Ours

Experiments

Geo. error (×100)	Train / Test	FAUST	SCAPE	FAUST SHREC19	FAUST TOPKIDS	FAUST DT4D-M	SCAPE	SCAPE FAUST	SCAPE SHREC19	SCAPE TOPKIDS	SCAPE DT4D-M
HKS [55]		43.0	-	-	-	-	40.5	-	-	-	-
CQHB-HKS [26]		37.2	-	-	-	-	31.6	-	-	-	-
CQHB-GT [26]		10.5	-	-	-	-	10.8	-	-	-	-
SyNoRiM(S) [23]		7.9	21.9	25.5	-	-	9.5	24.6	26.8	-	-
GeomFMaps(S) [14]		6.1	11.2	10.8	26.2	38.5	7.7	9.0	12.4	21.7	28.6
WSupFMNet(W) [50]		6.0	12.5	13.8	28.9	40.2	11.3	7.5	12.6	24.5	30.1
DiffFMaps(S) [33]		4.3	18.7	14.6	20.5	18.5	14.4	10.8	14.2	18.0	15.9
NIE(W) [25]		5.9	16.7	15.1	18.9	13.3	11.6	8.6	13.2	16.2	12.1
SSMSM(W) [7]		**2.4**	**6.8**	**9.0**	14.2	**11.8**	4.1	4.1	**5.2**	12.3	8.0
Ours(W)		3.7	8.7	9.5	**13.7**	13.1	**3.2**	**3.7**	8.1	**11.0**	**7.8**



References

[1] Sharp, Attaiki, Crane, Ovsjanikov. DiffusionNet: Discretization Agnostic Learning on Surfaces. ACM TOG 2022.
[2] Kovnatsky, M. Bronstein, M. Bronstein, Glashoff, Kimmel. Coupled Quasi-harmonic Bases. CGF 2013.
[3] Levy, Zhang. Spectral Geometry Processing. ACM SIGGRAPH ASIA Course Notes, 2009.
[4] Marin, Rakotosaona, Melzi, Ovsjanikov. Correspondence Learning via Linearly-invariant Embedding. NeurIPS 2020.
[5] Jiang, Sun, Huang. Neural Intrinsic Embedding for Non-rigid Point Cloud Matching. CVPR 2023.
[6] Cao, Bernard. Self-Supervised Learning for Multimodal Non-Rigid 3D Shape Matching. CVPR 2023.